1's Complement & 2's Complement

Representing signed integers can be done in 3 different schemes:

1. Sign Magnitude
2. 1's complement &
3. 2's complement

MSB → is the sign bit.

0 as MSB → it is a +ve integer

1 as MSB →it is a –ve integer

1's Complement

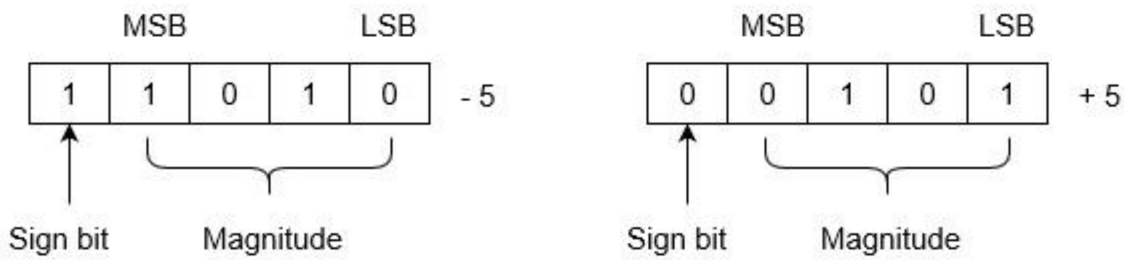
To get 1's complement of a binary number, simply invert the given number.

Binary number	1's complement
000	111
001	110
010	101
011	100
100	011
101	010
110	001
111	000

In the case of negative binary number representation, we represent in 1's complement.

First represent the number with positive sign and then take 1's complement of that number.

Example: Let we are using 5 bits register. The representation of -5 and +5 will be as follows:



+5 is represented as it is represented in sign magnitude method.

-5 is represented using the following steps:

(i) +5 = 0 0101

(ii) Take 1's complement of 0 0101 and that is 1 1010. **MSB** is 1 which indicates that number is negative.

MSB is always 1 in case of negative numbers.

Example-1: 10101110

01010001

Example-2: 10001.001

01110.110

Example- 3: -13 in 8 bit representation

Binary equivalent of +13 → 0000 1101

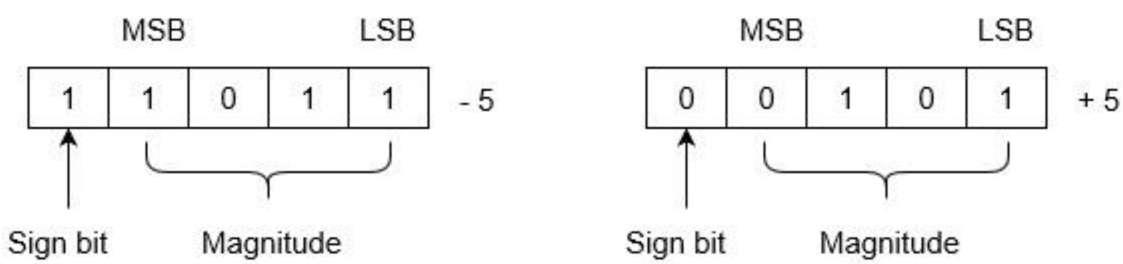
1's complement of +13 → 1111 0010 → -13

  1111 1111
- 0000 1101 → +13
  1111 0010 → -13

2's Complement

To get 2's complement of binary number take1's complement of given number and plus 1 to the least significant bit **(LSB).**

Let we are using 5 bits registers. The representation of -5 and +5 will be as follows:



+5 is represented as it is represented in sign magnitude method. -5 is represented using the following steps:

(i) +5 = 0 0101

(ii) Take 2's complement of 0 0101(1's complement→1 1010) and that is 1 1011. MSB is 1 which indicates that number is negative.

MSB is always 1 in case of negative numbers.

Example- 1: -13 in 5 bit representation

0 1101 → +13

= 1 0010 → (1's complement)

+ 1

= 1 0011 → (2's complement)

Example- 2: -17 in a 8 bit registry

0001 0001 → +17

= 1110 1110 → (1's complement)

+ 1

= 1110 1111 → (2's complement)

Addition & Subtraction using 1's complement notation

Subtractions by 1's Complement:

- Take 1's complement of the subtrahend.
- Add with minuend.
- If the result of above addition has carry bit 1, then add it to the least significant bit (LSB) of given result.
- If there is no carry bit 1, then take 1's complement of the result which will be negative.

Case-1: When Carry bit 1

Evaluate 10101 - 00101

1's complement of subtrahend : 00101 is
11010

Now, 10101

$$
\begin{array}{rr}
+ & 11010 \\
\hline
= & 1\ 01111 \\
+ & 1 \\
\hline
= & 10000
\end{array}
$$

1’s complement of 10000 is 01111

Case-2: When no Carry bit:

Evaluate 11110 with **11101**

1’s complement of subtrahend, 11110 is

00011

Now,

$$
\begin{array}{rr}
 & 11001 \\
+ & 00011 \\
\hline
= & 11100\ .
\end{array}
$$

Since there is no carry bit 1, so take 1’s complement of above result, which will be 00011 and i.e, 00011 is the answer.

<u>Additions by 1’s Complement:</u>

Case-1: Addition of positive and negative number when positive number has greater magnitude:

- Find out 1’s complement of negative number
- The end-around carry of the sum is added to the least significant bit (LSB).

Example: Add 1110 and -1101.

1’s complement of 11101 is

$$
\begin{array}{lr}
 & 10010 \\
\text{Now, add} & 01110 \\
+ & \underline{10010} \\
= & 1\,00000 \\
+ & \underline{1} \\
= & 00001
\end{array}
$$

Case-2: Addition of positive and negative number when negative number has greater magnitude:

- Find out 1's complement of negative number
- Add with given positive number.
- There will not be any end-around carry bit, take 1's complement of the result and this result will be negative.

Example: Add 1010 and -1100 in five-bit registers.

Five-bit registers, so it will be 01010 and 11100.

$$
\begin{array}{lr}
\text{1's complement of} & 1100 \text{ is} \\
 & 10011 \\
\text{Now, add} & 01010 \\
+ & \underline{10011} \\
= & 11101\,.
\end{array}
$$

Then take 1's complement of this result, which will be 00010 and this will be negative number, i.e., -00010, which is the answer.

Case-3: Addition of two negative numbers:

- Find out 1's complement for both numbers
- Add these 1's complement of numbers.
- There will always be end-around carry bit. Add this again to the LSB of result.
- Now, take 1's complement also of previous result, and this will be a negative number.

Example: Add -1010 and -0101 in five bit-register.

Five bit numbers,

So, -1010 → 11010 and

-00101→10101

1's complement of	11010	is	10101
1's complement of	10101	is	11010

Now, add

$$
\begin{array}{rr}
 & 10101 \\
+ & 11010 \\
\hline
= & 1\ 01111 \\
+ & 1 \\
\hline
= & 10000\ .
\end{array}
$$

Now take the 1's complement of this result, which will be 01111 and this number is negative, i.e, -01111, which is answer.

2's complement

Subtractions by 2's Complement

- Take 2's complement of the subtrahend
- Add with minuend
- If the result of above addition has carry bit 1, then it is dropped and this result will be positive number.
- If there is no carry bit 1, then take 2's complement of the result which will be negative
- Note that subtrahend is number that to be subtracted from the another number, i.e., minuend.

(Note that adding end-around carry-bit occurs only in 1's complement arithmetic operations but not 2's complement arithmetic operations)

Case-1: When Carry bit 1

Evaluate 10101 - 00101

So, 1's complement of subtrahend 00101 → 11010

2's complement of subtrahend 00101 → 11011

Now, add

$$
\begin{array}{r}
10101 \\
+\quad 11011 \\
\hline
=\quad 1\ 10000.
\end{array}
$$

Since, there is carry bit 1, so dropped this carry bit 1, and take this result will be 10000 will be positive number.

Case-2: When no Carry bit

Evaluate

10110 – 11010

Solution:

2's complement of 11010 is (00101 + 1) i.e. 00110. Hence

Minued -	1 0 1 1 0
2's complement of subtrahend -	+ 0 0 1 1 0
Result of addition -	1 1 1 0 0

As there is no carry over, the result of subtraction is negative and is obtained by writing the 2's complement of 11100 i.e.(00011 + 1) or 00100.

Hence the difference is – 100.

Additions by 2's Complement −

Case-1 − Addition of positive and negative number when positive number has greater magnitude:

- Find 2's complement of negative number.
- Carry bit 1 is dropped and this result will be positive number.

Example

Add 1110 and -1101.

2's complement of 1101 is

0011

Now, add	1110
	+ 0011

= 1 0001

Carry bit 1 is dropped and this result will be positive number, i.e., +0001.

Case-2 − Addition of positive and negative number when negative number has greater magnitude −

- Take 2's complement of negative number
- And add with given positive number.
- There will not be any end-around carry bit.
- Take 2's complement of the result and this result will be negative.

Example

Add 1010 and -1100 in five-bit registers.

Five bit register → 01010 & 11100

2's complement of 11100 →10100

Now, add 01010

\+ 10100

= 11110

Then take 2's complement of this result, which will be 00010 and this will be negative number, i.e., -00010, which is the answer.

Case-3 − Addition of two negative numbers −

- Take 2's complement for both numbers
- Add these 2's complement of numbers.
- Since there will always be end-around carry bit, so it is dropped.

- Now, take 2's complement also of previous result, so this will be negative number.

Example –

Add -1010 and -0101 in five bit-register.

Five bit register → 11010 & 10101

2's complement are 10110 &11011

Now, add 10110

$$
\begin{array}{rr}
 & 10110 \\
+ & \underline{11011} \\
= & 1\ 10001
\end{array}
$$

Since, there is a carry bit 1, so it is dropped.

Now take the 2's complement of this result, which will be 01111 and this number is negative, i.e, -01111, which is answer.